AD EDUXIAN JOURNAL
A Quarterly Multidisciplinary Blind Peer Reviewed & Refereed Online International Journal
Google Scholar Indexing PIF: 1.048(I2OR), 3.125 (IIFS) & 3.8(SJIF)
Vol (3), Issue (2), April-June 2026 ISSN: 3048-7951

Fuzzy Interior Ideals in Ternary Semigroups: Structural Properties and Characterizations

Dr.Thiriveedi Sunitha[1]

DOI: https://doi.org/10.5281/zenodo.20437712

Review: 01/05/2026 **Acceptance: 04/05/2026** **Publication: 29/05/2026**

ABSTRACT

A ternary semigroup is an algebraic structure equipped with a ternary associative operation, and interior ideals play a crucial role in understanding its substructures. By introducing fuzzy sets into this framework, we define fuzzy interior ideals and investigate their fundamental properties. In this paper we introduce the notion of fuzzy interior ideals in Ternary semigroups and investigated relations between fuzzy ideals and fuzzy interior ideals in ternary of regularity. We finally introduce the concept of a fuzzy simple ordered semigroup, we proved that an ordered ternary semigroup is simple if and only if it is fuzzy simple, and we characterize the simple ordered ternary semigroups in terms of fuzzy interior ideals.

Key words: Ternary semigroup, Fuzzy interior ideal, Fuzzy ideals.

1. INTRODUCTION

Zadeh introduced the concept of fuzzy sets and fuzzy set operations .After that several authors explored on the generalization of the notion of fuzzy set. This paper explores the concept of **fuzzy interior ideals** in **ternary semigroups**, an extension of classical interior ideals through the integration of fuzzy set theory. A ternary semigroup is an algebraic structure equipped with a ternary associative operation, and interior ideals play a crucial role in understanding its substructures. By introducing fuzzy sets into this framework, we define fuzzy interior ideals and investigate their fundamental properties. Specifically, we establish necessary and sufficient conditions for a fuzzy subset to be an interior ideal and examine its stability under ternary operations.

2. PRELIMINARIES

2.1. Definition: A non-empty set Z is said to be ternary semigroup if there exists a ternary operation. $: Z \times Z \times Z \to Z$ written as $(a,b,c) \to a.b.c$ satisfies the following identity $ab(cde) = a(bcd)e = (abc)de$ for any, $b, c, d, e \in Z$.

Example: Let Z be a set of all non -positive integers without zero. Then Z is a ternary semigroup, where the multiplication of a, b, c is denoted by abc where a, b, $c \in Z$.

2.2. Definition: Fuzzy subset of a non empty set is a collection of objects .Each object is assigned a value between 0 and 1 by a membership function.

2.3. Definition: Let X is a non empty set. A fuzzy set μ of the set X is a function. $\mu : X \to [0,1]$.

2.4. Definition: Let Y is a semigroup. A map A from Z to [0,1] is called a fuzzy set in Z. Let $F(Z)$ denote the set of all fuzzy sets in Z. For $P, Q, R \in F(Z)$, $P \subseteq Q$ and $Q \subseteq R$ if and only if $P(x) \le Q(x)$ and $Q(x) \le R(x)$ in the ordering of [0,1], for all $x \in Z$

2.5. Definition: A fuzzy set $A \in F(Z)$ is said to be a fuzzy sub semi-group of Z if $A(xyz) \geq \min\{A(x), A(y), A(z)\}$, for all $x, y, z \in Z$

[1] Lecturer in Mathematics, K V R GOVT Degree college for Women, Cluster University, R S Road, Kurnool, Andhra Pradesh, India.

2.6. Definition: A fuzzy set $A \in F(Z)$ is said to be a fuzzy left (resp., lateral and right) ideal of Z if $A(xyz) \geq$ A(z), (resp., $A(xyz) \geq A(x)$, and $A(xyz) \geq A(y)$) for all $x, y, z \in Z$.

2.7. Definition: A fuzzy sub semi group $A \in F(Z)$ is said to be A fuzzy bi ideal of Z if $A(xsytz) \geq \min\{A(x), A(y), A(z)\}$, for all $x, s, y, t, z \in Z$.

2.8. Definition: If Z is a ternary semigroup, a nonempty subset A of Z is called an interior ideal of Z if $Z\,Z\,A\,Z\,Z \subseteq A$.

2.9. Definition: Let Z be a ternary semigroup. A fuzzy subset f of Z is called a fuzzy interior ideal of Z, if $f(xuavy) \geq f(a)$ for all $x, u, a, v, y \in Z$.

2.10. Definition: A ternary semigroup Z is called left (right, lateral) simple if for every left (right, lateral) ideal A of Z, we have $A = Z$.

2.11. Lemma: Let Z be a ternary semigroup. Then Z is left (right, lateral) simple if and only if $ZZa = Z(aZZ = Z \text{ or } ZaZ = Z)$ for all $a \in Z$.

2.12. Definition: Let Z be a ternary semigroup. Z is called fuzzy simple if every fuzzy ideal of Z is a constant function, that is, for all $c, d \in Z$ we have $f(c) = f(d)$, where f is a fuzzy ideal Z.

2.13. Definition: Let f be any fuzzy subset of a ternary semigroup Z. The set

$f_t = \{x \in T \,/\, f(x) \geq t\}$, where $t \in [0, 1]$ is called a level subset of f.

2.14. Notation: If Z is a ternary semigroup and $c \in Z$, we denote by T_c the subset of Z define as follows:

$$T_c = \{d \in Z \,/ f(d) \geq f(c)\}.$$

3. MAIN RESULTS

3.1. Theorem: If Z is a ternary semigroup and f is a fuzzy ideal of Z . Then f is a fuzzy interior ideal of Z.

Proof: Let Z be a ternary semigroup and let f be a fuzzy ideal of Z.

To prove that f is a fuzzy interior ideal of Z.

Let $x, u, a, v, y \in Z$.

Since f is a fuzzy ideal of Z implies f is left, right lateral fuzzy ideal of Z .

Consider $f(xu(avy)) \geq f(avy)$ and $f(avy) \geq f(a)$ implies$f(xuavy) \geq f(a)$.

Hence f is fuzzy interior ideal of Z.

3.2. Theorem : If Z is a ternary semigroup and B is a nonempty subset of Z. Then B is an interior ideal of Z if and only if the characteristic function f_B is a fuzzy interior ideal of Z.

Proof: Let $, u, b, v, y \in Z$. If b ∈ B, then $f_B(b) := 1$. Since B is an interior ideal of Z, we have $xubvy \in$ ZZBZZ ⊆ B. Since $xubvy \in$ B,
we have $f_B\ (xubvy) := 1$. Then we have$f_B\ (xubvy) \geq f_B$ (b).
Hence f_B is a fuzzy interior ideal of Z.
Let $x, u, b, v, y \in Z$, b ∈ B. Since f_B is a fuzzy interior ideal of Z, we have $f_B\ (xubvy) \geq f_B$ (b). Since b ∈ B, we have f_B (b) := 1, so $f_B\ (xubvy) \geq 1$. Then $f_B\ (xubvy) = 1$, and xby ∈ B. Hence we have ZZBZZ ⊆ B.

Hence B is an interior ideal of Z.

3.3.Theorem: If Z is a ternary semigroup and f is a fuzzy right ideal of Z. Then the set T_c is a right ideal of Z for every c ∈ Z.

Proof: Let Z be a ternary semigroup and f be a fuzzy right ideal of Z.

Since T_c is a non empty subset of Z for any c ∈ Z. (since c ∈ T_c).

Let $d \in T_c$ and $p, q \in$ Z. Then $dpq \in T_c$.

Since f is a fuzzy right ideal of Z and $d, p, q \in$ Z, we have $f(dpq) \geq f(d)$.

Since $d \in T_c$, we have $f(d) \geq f(c)$. Then $f(dpq) \geq f(c)$, so $dpq \in T_c$.

Let $d \in T_c$ and $d \in Z$ such that $p \leq d$. Then $p \in T_c$.

Since f is a fuzzy right ideal of Z, $d, p \in$ Z and $p \leq d$, we have $f(p) \geq f(d)$.

Since $d \in T_c$ and $f(d) \geq f(c)$ we have $f(p) \geq f(c)$, so $p \in T_c$.

Hence the set T_c is a right ideal of Z .

Similarly we can prove

3.4. Theorem : Let Z be a ternary semigroup and f a fuzzy left ideal of Z. Then the set T_c is a left ideal of Z for every a ∈ Z.

3.5.Theorem : Let Z be a ternary semigroup and f a fuzzy lateral ideal of Z . Then the set T_c is a lateral ideal of Z for every a ∈ Z.

3.6. Lemma : Let Z be a ternary semigroup and f a fuzzy ideal of Z. Then the set T_c is an ideal of Z for every a ∈ Z.

3.7. Lemma : Let Z be a ternary semigroup and a non empty set D ⊆Z. Then D is an ideal of Z if and only if the characteristic function f_D is a fuzzy ideal of Z.

3.8. Theorem : A ternary semigroup Z is simple if and only if it is fuzzy simple.

Proof: Let Z be a simple ternary semigroup. then I = Z for any ideal I of Z.

To prove that Z is fuzzy simple. i.e., for every fuzzy ideal f of Z,

we have $f(c) = f(d), \forall c, d \in Z)$.

Let f be a fuzzy ideal of Z and $c, d \in Z$.

Since f is a fuzzy ideal of Z and $c \in Z$, by **Lemma 3.6**, the set T_c is an ideal of Z.

Since Z is simple, we have T_c = Z. Then d ∈ T_c, from which $f(d) \geq f(c)$.

By symmetry, we get $f(c) \geq f(d)$.

Therefore $f(c) = f(d)$ and Z is fuzzy simple.

Conversely,

Let Z be a fuzzy simple.

Implies for every fuzzy ideal f of Z, we have $f(c) = f(d), \forall c, d \in Z)$

To prove that Z is a simple ternary semigroup.

i.e., I = Z for any ideal I of Z.

Suppose Z contains proper ideals and let D be an ideal of Z such that $D \neq Z$.

By **Lemma 3.7**, f_D is a fuzzy ideal of Z. We have $D \subseteq Z$.

Let x ∈ Z. Since Z is fuzzy simple, the fuzzy ideal f_D is a constant function,

that is, f_D (x) = f_D (b) for every b ∈ Z.

now for any a ∈ D ($D \neq \varphi$). we have f_D (x) = f_D (a) := 1,

Hence x ∈ D. Thus we have $Z \subseteq D$, and so $Z = D$.

Which is contradiction. Therefore Z contains no proper ideals.

3.9. Lemma: A ternary semigroup Z is simple if and only if for every a ∈ Z, we have $Z = ZZaZZ$.

3.10. Theorem : Let Z be a ternary semigroup. Then Z is simple if and only if every fuzzy interior ideal of Z is a constant function.

Proof: Let Z be a simple ternary semigroup.

We have to prove that every fuzzy interior ideal of Z is a constant function.

Let f be a fuzzy interior ideal of Z and $c, d \in Z$.

Since Z is simple and $d \in Z$ by **Lemma 3.9**, we have $Z = ZZdZZ$.

Since $\in Z$, we have $c \in ZZdZZ$. Then there exist $x, u, v, y \in Z$ such that $c \leq xudvy$.

Since $c, xudvy \in Z, c \leq xudvy$ and f a fuzzy interior ideal of Z, we have $f(c) \geq f(xudvy)$.

Since $x, u, d, v, y \in Z$ and f is a fuzzy interior ideal of Z, we have $f(xudvy) \geq f(d)$. Then we have $f(c) \geq f(d)$ Similar way we can prove $f(c) \leq f(d)$.

Implies $f(c) = f(d)$ and hence f is a constant function.

Let f be a fuzzy ideal of Z. By **Theorem 3.1**, f is a fuzzy interior ideal of Z.

By hypothesis, f is a constant function. Then Z is fuzzy simple and, by **Theorem 3.8**, Z is simple.

3.11. Theorem: For a any ternary semigroup Z, the following are equivalent:

1) Z is simple.
2) $Z = ZZaZZ$ for every a ∈ Z.
3) Z is fuzzy simple.
4) Every fuzzy interior ideal of Z is a constant function.

Proof: It follows by **theorem 3.8; Lemma 3.9; and theorem 3.10**.

4. CONCLUSION

Interior ideals in ternary semigroups generalize the concept of ideals in classical semigroup theory. They provide insight into the internal structure of ternary semigroups and have both theoretical and practical applications. Further research includes their relationship with other substructures like bi-ideals and quasi-ideals in ternary semigroups.

5. REFERENCES

1 .Zadeh, L.A.Fuzzy sets, Inform. Control, 8, pp. 338 – 353.

1. Sison, F.M. Ideal theory in ternary semigroups", Math.Japon,10(1965),63-84

2. Kehayopulu N. and Tsingelis, M. Regular oerderd semigroups in terms of fuzzy subsets. information science, 176 (2006) 3675 – 3693.

3. Dixit V.N. and Dewan S.A note on quasi and bi ideals in ternary semigroups, international journal of Mathematical Society, 18(3) (199), 501-508.

4. T Sunitha, Dr. U. Nagi Reddy, Dr. G. Shobhalatha,.and C. Meena Kumari.:Ternary semigroups T Satisfying the identity b aba= for all T aba ∈ T, , Journal of Computer and Mathematical 11 Sciences, Vol 10(4),780-786April 2019, ISSN 2319 8133.

5. Sunitha T., Shobhalatha G. and Nagi Reddy U., Some Properties Of Simple Ternary Semigroups,International Journal of Mathematics Trends and Technology (IJMTT) Â Volume 65 Issue 6, ISSN:2231-5373, June 2019.

6. T. Sunitha, Dr, G, Shobhalatha ,Strongly Regular Ternary semigroups, Mathematical Sciences International Research Journal Volume 9 Issue 2,61-68,Aug 2020. ISSN 2278- 8697

7. U. Nagi Reddy and G. Shobhalatha, Some Characterizations Of Ternary Semigroups, September –2015, ISSN 2229-5518 IJSER © 2015 .http://www.ijser.org.

8. U. Nagi Reddy, U. Meena Kumari, C.and Shobhalatha, G. Some Properties of Fuzzy Quasi ideals in Ternary Semigroups, International Journal of Development Research, Volume 7 Issue 12 Pages 17512-17518, December - 2017.

9. T Sunitha1, U Nagi Reddy, G Shobhalatha, A Note on Full k-Ideals in Ternary Semirings, Indian Journal of Science and Technology 2021;14(21):1786–1790